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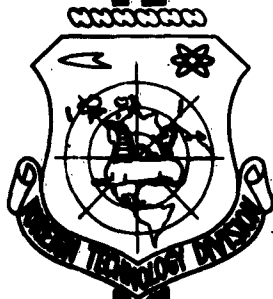
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(SELECTED ARTICLES)

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AIR FORCE SYSTEMS COMMAND

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TURBULENT BOUNDARY LAYER WITH PRESSURE GRADIENTS

ON A POROUS SURFACE

A.I. Tolstykh

(Moscow)

Existing methods of calculating a turbulent boundary layer with the transport of a substance through the surface are restricted only by the case of zero pressure gradients and are based on the use of the same semiempirical relationships as in the case of an impermeable plate [1]-[4], [8], [9].* Because of the lack of the required experimental data the latter unavoidably results in the assumption that certain quantities are independent of the mass-transfer parameters and to a somewhat arbitrary approach in the selection of the necessary constants.

The purpose of the present work is to investigate the more general case, i.e., a turbulent layer on a porous surface in the presence of pressure gradients, without the introduction of any semiempirical relationships and with the utilization of only that information which can be obtained from the known prior history of the flow.

Initially an incompressible boundary layer is considered in the absence of diffusion, and then a formal generalization is carried out for the case of a heterogeneous mixture of compressible gas.

1. To derive the required relationships we will proceed from the case of an idealized two-layer model and will assume a turbulent layer consisting of a wall region in which the equations of laminar motion

and a turbulent core are valid. In accordance with this, one part of the flow may be determined by assuming the required number of boundary conditions, and with respect to the other part of the flow (because the equations of the turbulent boundary layer are open) it becomes necessary to make certain assumptions.

Flow in the wall region (a viscous sublayer) within the scope of the boundary-layer equations may be described by a system of the following form:

$$\begin{aligned} u \frac{\partial u}{\partial x} + v_1 \frac{\partial u}{\partial y} &= \sum Q_i + \nu \frac{\partial^2 u}{\partial y^2} \\ \frac{\partial u}{\partial x} + \frac{\partial v_1}{\partial y} &= 0 \end{aligned} \quad (1)$$

with boundary conditions at $y = 0$

$$u=0, \quad v_1=0, \quad \frac{\partial u}{\partial y} = \frac{\tau_w}{\nu_p}, \quad Q_i = Q_{iw}.$$

Here $v_1 = v - w$; u and v are velocity components in the conventional boundary-layer coordinates x and y ; w is the normal velocity at the surface; τ_w , ρ , and ν is the stress at the wall, and the density and kinematic viscosity; Q_1 and Q_{1w} are, respectively, certain functions characterizing the flow and their value at the wall. For example, in the presence of a pressure gradient $\sum Q_1 = Q_1 = -(1/\rho)(\partial x/\partial p)$, as a gas is passed through the surface $\sum Q_1 = Q_2 = w(\partial u/\partial y)$, etc.

At given values of τ_w and Q_{1w} System (1) fully determines the motion near the wall and in view of the smallness of the nonlinear members is reduced to the following equation:

$$\nu \frac{\partial^2 u}{\partial y^2} + \sum Q_i = 0$$

with the corresponding boundary conditions. From the concept of dimensionality it follows that the solution to the above equation has the following form:

$$u = v_* q \left(\frac{y^2}{v_*^2}, \frac{Q_{1w} v}{v_*^3} \right),$$

where $v_* = \sqrt{(\tau_w/\rho)}$ is the dynamic velocity.

In particular, when air is forced through a surface along which there is the following pressure gradient:

$$q = \frac{a^* + f^*}{f^*} (e^{f^* y_*} - 1) - \frac{a^*}{f^*} y_*,$$

where $y_* = (y v_*/v)$, and a^* and f^* are the dimensionless parameters of the pressure gradient and the air injection:

$$\begin{aligned} a^* &= \frac{1}{\rho} \frac{\partial p}{\partial x} \frac{v}{v_*^3}, \\ f^* &= \frac{w}{v}. \end{aligned} \quad (2)$$

The wall boundary layer in which the flow is described by a "laminar" solution is naturally defined as a line along which laminar friction covers a definite and fully established portion of the turbulent layer; let us say that it is defined as the line along which $v_T/v = \text{const}$, where v_T is the turbulent (virtual) viscosity that is associated with turbulent friction by the relationship $v_T = \rho v_T (\partial u / \partial y)$. This representation, accurate to the constant, defines the "laminar sublayer" on the impermeable plate, since the relationship v_T/v must be a function only of y_* . In the general case, this relationship may be a function also of the parameters Q_{1w} . However, if instead of τ_w the local value of total friction τ is taken as one of the determining parameters, the effect of Q_{1w} must make itself felt, apparently, only by changing the value of τ . Indeed, on the one hand $\tau = \rho v \left(1 + \frac{v_T}{v} \right) \frac{\partial u}{\partial y}$, while on the other hand, as a result of evaluating the various terms in the balanced equation for pulsating energy [10], $(v_T/v) (\partial u / \partial y)^2 \approx \varepsilon$ (ε denotes those terms which correspond to the averaged viscous dissipation of pulsation), only if convection transfer of the pulsating en-

ergy by the lateral velocities is not too great (if the injection velocity w is not too great) and y_* is not too small.

In view of the fact that ϵ is not an explicit function of Q_{1w} and is most likely determined by τ (by changing the velocity scale for major pulsations), the ratio v_T/v is a function only of y , τ , ρ , and v , i.e., the condition $v_T/T = \text{const}$ is equivalent to the relationship $(y/v)\sqrt{(\tau/\rho)} = \text{const}$. Since in an idealized two-layer model the total stress near the wall is only viscous, we require that at the boundary of the viscous sublayer the following relationship be satisfied:

$$\frac{y}{v} \sqrt{v \frac{\partial v_x q}{\partial y}} = \text{const}, \quad (3)$$

where the constant in the right-hand part is independent of the x coordinate and, consequently, it is independent of the parameters of injection and the gradient. This constant can be determined if the required magnitudes pertaining to some "initial" section are known; in particular, if the boundary layer on the impermeable plate has been thoroughly studied in the prior history of the given flow, $\text{const} = \alpha$, where α is a universal constant. In view of the universality of α we can maintain that the following is valid regardless of the existence of such a prior history:

$$\lim_{\substack{q^* \rightarrow 0 \\ \rho^* \rightarrow 0}} \frac{1}{v} y \sqrt{v \frac{\partial v_x q}{\partial y}} = \text{const} = \alpha,$$

Thus Condition (3) can be written in the following form:

$$Y_* \frac{\partial q}{\partial y} \Big|_{y=y_*} = Y_*^2, \quad (4)$$

where Y is the value of y_* at the boundary of the viscous region, and $Y_0 = \alpha$.

Let us now examine the turbulent portion of the boundary layer. Restricting the classes of flows being studied somewhat, we will as-

sume that the velocity profile in any section can be presented in the "single-parameter" form

$$\frac{u}{u_e} = f\left[\frac{y}{\delta}, r(Q_{1w})\right], \quad f(1, r, (Q_{1w})) = 1, \quad (5)$$

where δ is the thickness of the boundary layer; u_e is the velocity at the outer limit of the boundary layer; r is some parameter. In the "initial" section the velocity distribution must, thus, be described by a function of the form of (5) at some initial value of r . In order to clarify the nature of the function $r(Q_{1w})$ we will use the concepts of dimensionality. The only magnitudes determining the flow in the vicinity of the boundaries of these regions will, evidently, be the total momentum flux τ at this boundary, density ρ , and the distance y ; from these parameters we can make only the single combination having the dimension of the derivative $\partial u / \partial y$, and namely $\frac{\partial u}{\partial y} = \text{const} \frac{\sqrt{\frac{\tau}{\rho}}}{y}$, and therefore at the boundary as a consequence of Relationship (3) and the condition of momentum-flux continuity we find the following equality

$$\frac{\partial u}{\partial y} = \text{const} \frac{\partial v_x q}{\partial y} \Big|_{y=0}, \quad (6)$$

which is accurate to the higher derivatives of the velocity u and yields the function $r(Q_{1w})$.

With respect to the form of the function f let us note that Formula (5) must determine the velocity profile even in the case in which the "initial" section corresponds to the flow on the impermeable plate; it is evident that Assumption (5) will be satisfied if the corresponding functional relationship will have the same form as in the initial section, i.e., for example,

$$\frac{u - u_e}{u_e} = \frac{1}{k(Q_{1w})} \ln \frac{y}{\delta} \quad \text{or} \quad \frac{u}{u_e} = \left(\frac{y}{\delta}\right)^{m(Q_{1w})}, \quad (7)$$

where the parameter r , denoted, respectively, by k and m , assumes the

known values $k(0) = k_0$ and $m(0) = m_0$.

As shown by the experimental data, the second of the functions (7) describes an extremely extensive class of flows (in particular, flows with nonpositive pressure gradients and flows with the movement, generally speaking, of a foreign gas through the porous surface of the plate [5-7]). It is good that the step distribution of velocity is preserved to substantial injection intensities [5], [7]. Subsequently we will use the step function which results in comparatively simple finite results; the exponential values will be found from Relationship [6], in which the constant, as a result of the universality of the initial value of m_0 , is determined as a result of the limit transition for $Q_{1w} \rightarrow 0$:

$$m = m_0 Y \left(\frac{1}{q} \frac{\partial q}{\partial y_*} \right)_{y_*=Y} \quad (8)$$

In the derivation of this formula we used the condition of velocity continuity which can also be written in the following form:

$$\sqrt{\frac{c_f}{2}} q \Big|_{y_*=Y} = \left(\frac{Y}{\sqrt{\frac{c_f}{2}} R} \right)^m \quad (9)$$

where $(c_f/2) = (v_*^2/u_e^2)$ and $R = (u_e \delta/\nu)$. Relationships (4), (8), (9), and (2) make it possible to calculate the local coefficient of friction and the exponent m , which when using the integral momentum relationship fully solves the problem.

Let us examine certain special cases.

a) Let, for example, $f^* = 0$, and $a^* < 0$ (the negative pressure gradient on the impermeable wall). In this case the velocity profile has a step shape, and Formulas (4) and (8) are transformed, respectively, to

$$Y^2(a^*Y + 1) = Y_*^2, \quad m = m_0 \frac{a^*Y + 1}{\frac{1}{2} a^*Y + 1}$$

These relationships yield the theoretical relationships between the coefficient of friction and the exponent and the pressure gradient, which are of some interest. Figure 1 shows these functions together with the experimental points pertaining to the coefficient of friction [12] in the following coordinates: c_f/c_{f0} , m/m_0 and a^* (the subscript 0 denotes the absence of a gradient).

b) Let $f^* > 0$ and $a^* = 0$ (injection on a porous plate). Then according to (4), (8), (9), and (2)

$$m = m_0 M(\bar{Y}),$$

$$\frac{c_f}{2} = T(\bar{Y}) Y_0^{-1} \frac{1-m}{1+m} R^{-1} \frac{m}{1+m},$$

where

$$\bar{Y} = \frac{Y}{Y_0}, \quad M(\bar{Y}) = 2 |\ln \bar{Y}| (1 - \bar{Y}^2)^{-1},$$

$$T(\bar{Y}) = \bar{Y}^2 [M(\bar{Y})]^{1+m}.$$

The values of \bar{Y} are found from the following equation:

$$\frac{w(x)}{u_*} = f^* \sqrt{\frac{c_f}{2}} = 2 |\ln \bar{Y}| [M(\bar{Y})]^{1+m} Y_0^{-1} \frac{1-m}{1+m} R^{-1} \frac{m}{1+m},$$

in which the left-hand part is assumed to be given.

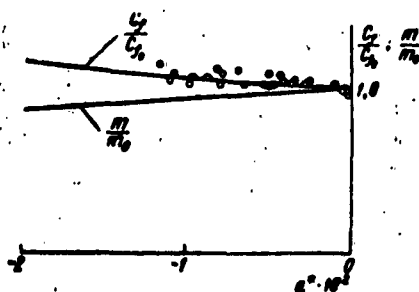


Fig. 1

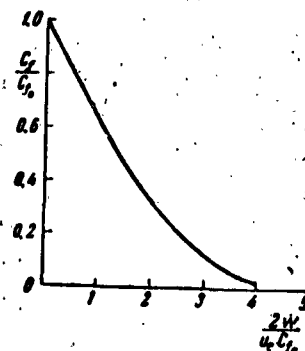


Fig. 2

The calculations that have been carried out showed that despite the absence of some degree of freely selected constants, the experimental values of \underline{m} in the most varied of cases are in satisfactory

agreement with the theoretical values, provided the injection velocity is not too great (at high injection velocities discontinuity phenomena are observed and the corresponding relationships do not accurately describe the velocity distribution); an analogous comparison of friction-coefficient values is difficult in view

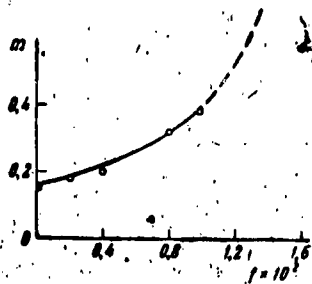


Fig. 3

of the substantial scattering of the experimental points. As an example, Figs. 2 and 3 show the data pertaining to Reference [5]; the curve in Fig. 2 has been constructed in a conventional system of coordinates c_f/c_{f0} and

$2w/u_e c_{f0}$ (the subscript 0 denotes the impermeable surface) and corresponds to the constant Reynolds number $R_x = (u_e x/\nu)$.

c) Let $f^* > 0$ and $a^* < 0$ (injection on a surface streamlined by an accelerated flow). In this case the sought quantities can be presented in the form of $m = m_0 M(\bar{Y}; Z_1, Z_2)$,

$$\frac{c_l}{2} = T(\bar{Y}; Z_1, Z_2) Y_*^{-\frac{1-m}{1+m}} R^{-\frac{m}{1+m}},$$

where $M = Z_1(1 - \bar{Y}^2 - Z_2 \bar{Y})^{-1}$, $T = \bar{Y}^3 M^{\frac{1}{1+m}}$, $Z_1 = Y f^*$, $Z_2 = Y a^*$, and the values of \bar{Y} , Z_1 and Z_2 are determined from the following equations:

$$\frac{w(x)}{u_e} = f^* \left(\frac{c_l}{2} \right)^{\frac{1}{3}},$$

$$\frac{1}{\rho} \frac{\partial p}{\partial x} \frac{\nu}{u_e^3} = a^* \left(\frac{c_l}{2} \right)^{\frac{2}{3}}$$

and Condition (4). Solutions taking this form can be used, for example, in investigating flow in the vicinity of the critical point. Figure 4 shows the results obtained in the calculation of the friction coefficient for various local values of the pressure gradient in the case of the same Reynolds number $R = (u_e \delta/\nu) = 10^4$ (the subscript 0 denotes the zero values of the parameters f^* and a^*). It should be

pointed out that the values of the coefficient of friction, pertaining to the same R_x number (for example, to one and the same point on the surface), will increase substantially more intensely with an increase in $\partial p / \partial x$ than shown in Fig. 4 as a result of the reduction in the R number.

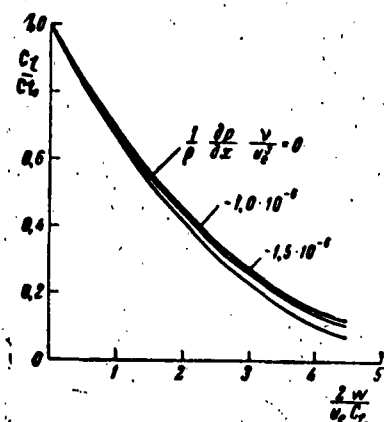


Fig. 4

2. The outlined concepts can be used to calculate the rather general case of a turbulent boundary layer in the presence of diffusion and heat transfer. The question is resolved particularly simply if we have a condition of similarity between the velocity, temperature, and concentration fields, i.e., if there is a "generalized" Crocco integral (derived, for example, in

Reference [8]). In the general case, if the functions governing the changes in the physical properties of the gas are known (viscosity, thermal conductivity, etc.), it is always possible to solve the system of equations which describes the flow near the wall (the equations in Reference [13] evaluated for low values of y); for this it is sufficient to pose the Koshi [sic] problem, assuming inadequate boundary conditions at the surface (for example, the magnitudes of friction, heat flow, and concentration). The solutions found with the aid of relationships of the type of (4), (8), and (9), and Assumption (5), if it actually is valid, make it possible to determine the wall region and the velocity profile in the turbulent part. With respect to the determination of the sought magnitudes on the surface we would point out that the equations of turbulent flow in a known velocity field and given known functions governing changes in "turbulent" Prandtl, Schmidt, etc. numbers, make it possible to associate the "ex-

ternal" boundary conditions with the conditions prevailing at the boundary of the viscous region; the latter, however, in view of the continuity of the corresponding magnitudes (velocity, temperature, concentration, and flows of momentum, heat, and matter) are expressed in terms of the boundary conditions at the wall.

As an example let us consider the flow of a binary mixture in a boundary layer on a porous surface with a nonzero negative pressure gradient; for the sake of simplicity we will assume that there is no heat transfer.* The solution for the viscous region in this case will be written in the following form:

$$q = \frac{a_1 + f^*}{f^{*2}} e^A \int_1^{f^{*2}} \exp(-A\xi^{Sc}) d\xi + \frac{a_1}{f^{*2} Sc} e^A [Ei(-Ae^{Sc} f^{*2}) - Ei(-A)], \quad (10)$$

$$1 + (\mu - 1)C = [1 + (\mu - 1)C_w] \exp[-A(e^{Sc} f^{*2} - 1)],$$

where $y = \frac{y v_*}{\nu}$, $f^* = \frac{w}{v_*}$, $v_* = \sqrt{\frac{\tau_w}{\rho_w}}$ (ρ_w is the density at the wall), Sc is the Schmidt number, $a_1 = \frac{1}{\rho_*} \frac{\partial p}{\partial x} \frac{\nu}{v_*^2} [1 + (\mu - 1)C_w]$ ($\rho_* = \rho|_{y=0}$) is the parameter of the pressure gradient, $A = (\mu - 1) \frac{1 - C_w}{1 + (\mu - 1)C_w}$ (C and C_w is the concentration and its value at the wall, $\mu = M_1/M_2$ is the ratio between the molecular weights of the basic and introduced gas). In the derivation of these expressions we made use of the balance equation for diffusion flows $\rho_w w (1 - C_w) = -\rho_w D_{12} \frac{\partial C}{\partial y} \Big|_{y=0}$ (D_{12} is the coefficient of diffusion); for the sake of simplicity it was assumed that $Sc = \nu/D_{12} = \text{const}$ and

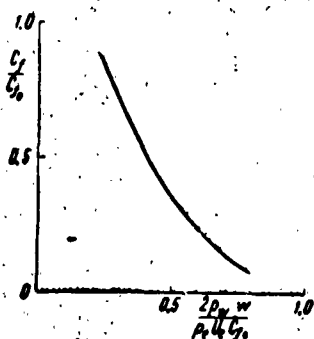


Fig. 5

as a result as well as in view of the fact that D_{12} calculated by the Enskog [11] method is a weak function of the mixture proportion, it was assumed that $\nu = \text{const}$.

In simplified terms, in the equations of turbulent motion we will assume $\partial/\partial x \equiv 0$ (for example, let this be as a first approxi-

mation); then, using the condition at the boundary of the regions, we will obtain

$$\begin{aligned} \rho v_T &= \frac{\rho_Y w u_e \bar{y}^m + \frac{\partial p}{\partial x} \delta \bar{y} - B}{m u_e \bar{y}^m} \delta \bar{y}, \\ \rho_Y w C &= \frac{1}{Sc_T} \rho v_T \frac{dC}{d\bar{y}} + D, \end{aligned} \quad (11)$$

where

$$\begin{aligned} \rho_Y &= \rho|_{y_e=y}, \quad B = v_e^2 \rho_Y \left(f' q - \frac{\partial q}{\partial y_e} + a_1 y_e \right) \Big|_{y_e=y}, \\ D &= v_e \rho_Y \left(C f' - \frac{1}{Sc} \frac{\partial C}{\partial y_e} \right) \Big|_{y_e=y}, \quad \bar{y} = \frac{y}{\delta}, \end{aligned}$$

$Sc_T = \text{const}$ is the turbulent Schmidt number.

Integrating the second of the equations in (11), we will find the concentration distribution which as a result of the condition $C = 0$ in the case of $y = \delta$ yields the relationship

$$C|_{y=y} = \frac{D}{\rho_Y w} \left[1 - e^{-\Phi(\bar{y})} \right], \quad (12)$$

where

$$\Phi(\bar{y}) = Sc_T \int_0^{\bar{y}} \frac{dt}{1 + \frac{\partial p}{\partial x} \frac{\delta}{\rho_Y w u_e} t^m - \frac{B}{\rho_Y w u_e}}.$$

For a given distribution of $w(x)$ and $u_e(x)$ Formulas (4), (8), (9), (10), and (12) make it possible to determine all of the unknown flow parameters contained in them. The results of the calculations carried out in the assumption that $\partial p / \partial x = 0$ and $Sc = Sc_T = 1$ for a mixture of air and helium are shown in Fig. 5. All of the values ($c_f/2 = \tau_w / \rho_e u_e^2$) pertain to the same Reynolds number $R = 10^4$. Figure 5 illustrates the sharp reduction in the coefficient of friction in the case of the injection of a light foreign gas; however, it should be pointed out that the final results of the relationship between the

physical properties of the mixture and the concentration ($D_{12} = \text{const}$, $\mu = \rho S c D_{12} = \rho \text{ const}$) that were assumed for the sake of simplicity result in a somewhat too rapid change in viscosity, and this contradicts the results of kinetic theory. Formally this indicates that with the introduction of light gases ($\mu > 1$) in the vicinity of $y_* f^* = 0$ the derivative $\partial^2 u / \partial y^2$ is always negative, in which connection, given small flow rates for the gas being introduced, there is observed the unlikely increase in the ratio c_f / c_{f0} .

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[Footnotes]

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- 1 In Reference [4] it has been suggested, however, that the obtained solutions be used to describe the flow in the vicinity of the critical point.
- 4 A special case of Relationship (4) was used in the work of Van Dreist [4].
- 6 The value of m_0 in view of Assumption (5) need not be a function of the Reynolds number, and for this reason Formula (8) does not take into account the insignificant evolution of the velocity profile in the case of constant parameter values (for example, on the impermeable plate).
- 7 The region of positive pressure gradients is not considered, since in this region, generally speaking, the assumption of a single-parameter velocity profile is not satisfied.
- 10 Qualitative evaluations have shown that in accordance with experimental data and various theoretical works the effect of heat transfer on c_f / c_{f0} and St / St_0 (St and St_0 are the Stanton numbers) as functions of the corresponding dimensionless mass flow rate (see Fig. 5) are generally not great.

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THE PROCESSES OF IONIZATION AND RECOMBINATION IN HOT AIR

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(Moscow)

In the passage of extremely powerful shock waves, not only molecular dissociation can take place in the air behind the shock-wave front, but the ionization of the atoms. An example of such a shock wave is the wave formed by a meteorite flying through the atmosphere (as is well known, the velocity of meteorites in the atmosphere ranges between 11 and 72 km/sec).

To calculate the processes of ionization and recombination in the hot air behind the front of a wave, the air in a plasma state, we must know the coefficients of ionization Z_r and recombination C_r (r is the degree of ionization). In order to obtain these coefficients we must evaluate the comparative role played by the various elementary processes, and in contemporary scientific literature this is, by no means, done in a uniquely defined manner.

1. Ionization. The basic mechanism of ionization in hot air is the impact of electrons. The general expression for the coefficient of ionization resulting from electron impact, as used in courses in astrophysics [1, 2], takes the following form:

$$Z_{r,k} = 4\pi \left(\frac{m_e}{2\pi k T_e} \right)^{\frac{3}{2}} \int_{v_0}^{\infty} \sigma_{r,k}(v) v^3 e^{-\frac{m_e v^2}{2k T_e}} dv. \quad (1)$$

Here v is the velocity of the colliding electron, v_0 is the minimum velocity resulting in ionization, $\sigma_{r,k}(v)$ is the effective cross

section of the r -th ionization from the k -th level, at velocity v .

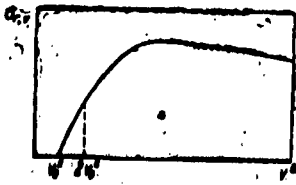


Fig. 1

In this case it is generally assumed that the ionization takes place primarily from the base level, since the population of excited levels in the case of the Boltzmann distribution is negligibly small. Let us try to calculate the

coefficient of ionization $Z_{r,1}$ from the base level. Generally [1, 2] the concept of the mean effective ionization cross section $\bar{\sigma}_{r,v}$ is used to calculate the integral in (1), placing this mean value outside of the radical sign. Then

$$Z_{r,1} = 2 \sqrt{\frac{2}{\pi}} \bar{\sigma}_{r,v} \left(\frac{kT_e}{m_e} \right)^{1/2} e^{-\frac{I_r}{kT_e}} \left(1 + \frac{I_r}{kT_e} \right) \quad (2)$$

Here it has been taken into consideration that $I_r = (m_e v_0^2 / 2)$. However, $\bar{\sigma}_{r,v}$ is a function of the temperature T_e and the ionization potential I_r , i.e., a function of v_0 . It is therefore more convenient, as was done by S.B. Pinkel'ner [3], to make use of the circumstance that in the energy integral of interest to us the quantity $\sigma_{r,v}$ increases almost linearly in the v^2 function (Fig. 1).

In this case

$$\sigma_{r,v} = \sigma_0 \frac{v^2 - v_0^2}{v_0^2}, \quad (3)$$

where σ_0 is the effective ionization cross section at an energy of $2I_r$, i.e., at a velocity of $v_0 \sqrt{2}$. Substituting (3) into (1) and calculating the integral, we will find the following expression for the ionization coefficient.*

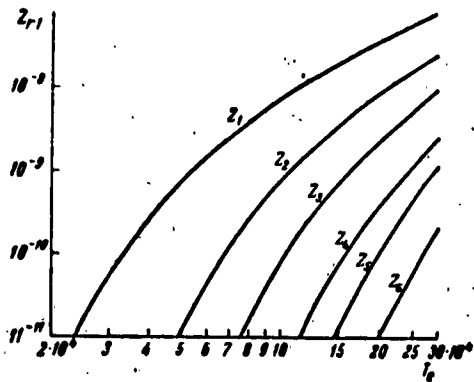
$$Z_{r,1} = 2 \sqrt{\frac{2}{\pi}} \sigma_0 \left(\frac{kT_e}{m_e} \right)^{1/2} e^{-\frac{I_r}{kT_e}} \left(1 + 2 \frac{kT_e}{I_r} \right). \quad (4)$$

From a comparison of (2) and (4) it follows directly that

$$\bar{g}_{r,s} = g_0 \frac{1 + \frac{2kT_e}{I_r}}{1 + \frac{kT_e}{I_r}} = g_0 \frac{kT_e}{I_r} \frac{I_r + 2kT_e}{I_r + kT_e} \quad (5)$$

Substituting the numerical values of the constants into Formula (4), we will bring it to a form convenient for calculations:

$$Z_{r,1} = 5.5 \cdot 10^{-11} T_e^{1/2} \left(1 + \frac{2kT_e}{I_r} \right) e^{-\frac{I_r}{kT_e}} \quad (4a)$$



The values of the coefficients of ionization from the base level, calculated for air according to Formula (4a) are presented in Fig. 2.*

However, the ionization from the base level by no means determines the general course of the ionization.

As has been shown by the Works of G.S.

Ivanov-Kholodnyy, G.M. Nikol'skiy,

Fig. 2

and R.A. Gulyayev [4, 5], as well as in the works of L.M. Biberman and his coworkers [6, 7], the chief contribution to ionization is made by the upper levels. Indeed, despite the fact that the population of these upper levels is small in comparison with the base level, the coefficient of ionization from the upper levels increases sharply as a result of the reduction in the binding energy of the levels.

In particular, the probability of ionizing hydrogen from the n level is equal [6] to

$$Z_n = 1.1 \cdot 10^{-11} T^{-1/2} n (-\text{Ei}(-u_n)), \quad (6)$$

where $u_n = (I_n/kTn^2)$, and $-\text{Ei}(-u_n)$ is the integral exponential function. The shape of the product of $n(-\text{Ei}(-u_n))$ with the number of the n level for various values of $u_1 = (I_1/kT)$ is presented in Fig. 3.

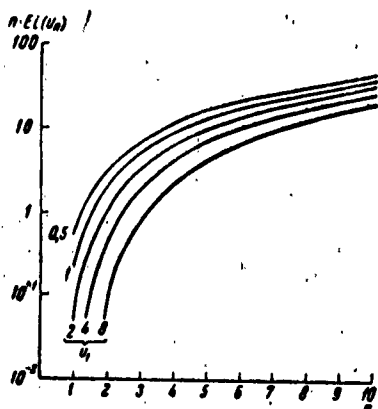


Fig. 3.

To determine the over-all coefficient of hydrogen ionization (from all levels) Z_n is summed in accordance with the Boltzman distribution:

$$Z = \frac{e^{-U_1}}{U(T)} \sum_n Z_n g_n e^{U_n} \quad (7)$$

where g_n is the statistical weight, $U(T)$ is the sum of the given atom or ion according

TABLE 1

1 Вид плазмы	n_e	T	n_0	Вид плазмы	n_e	T	n_0
2 Газовые туманности	10^4	10^4	870	Протуберанцы	10^{12}	$7,5 \cdot 10^4$	40
3 Ионосфера	10^6	10^3	420	Горячий воздух	10^{16}	10^3	8-10
4 Солнечная корона	10^8	10^6	190	То же	10^{20}	10^6	1-2
5 Хромосфера	10^{11}	$5 \cdot 10^6$	58				

1) Type of plasma; 2) gas nebulae; 3) ionosphere; 4) solar corona; 5) chromosphere; 6) protuberances; 7) hot air; 8) the same.

to state. Sum (7) exhibits a divergence if we do not take into consideration the deviation of the distribution over the levels from the Boltzmann distribution, as a result of the mutual excitation of the ions (the effects of preionization and supercharging). This phenomenon was examined in the classical Tomson [sic] approximation by G.S. Ivanov-Kholodnyy et al. [4] and in the Bethe-Born approximation by L. M. Biberman, Yu.N. Toropnik and K.N. Ul'yanov [6]. The qualitative results of these two works coincide and can be reduced to the fact that levels above sum n_0 in a real gas cannot be achieved. However, an evaluation of n_0 for hydrogen in Reference [4] is somewhat underestimated (i.e., the effect of supercharging is exaggerated), as is shown by a comparison against experimental data [6].

For purposes of evaluating n_0 Reference [4] proposes the follow-

ing formula (close to the analogous Unzold [sic] formula)

$$\log n_e = 21.65 - 6 \log (n_0 + 1), \quad (8)$$

where n_e is the electron concentration. For the various types of plasma in [4] the following values of n_0 have been found, and to these we have added the values for hot air (Table 1).

As can be seen, a comparatively small quantity of levels can actually be achieved in hot air. On the one hand this circumstance facilitates the summing and makes it possible to do entirely without the integration of Formula (7) with respect to the upper levels, as is generally done. However, the small number of levels imposes greater requirements upon the correct evaluation of n_0 .

L.M. Biberman and his coworkers [6] obtained the following approximate expression for Z_r (in the interval from $0.01 < u_0 < 5$, where $u_0 = I_r/kTn_0^2$) as a result of integrating Formula (6) with respect to the Boltzmann distribution ($0 < n < n_0$):

$$Z_r = 2.2 \cdot 10^{-8} T^{-1/2} e^{-u_0} \frac{u_0^5}{u_0^{5.6}}, \quad (9)$$

which can be transformed to

$$Z_r = 2.2 \cdot 10^{-8} T^{-1/2} e^{-\frac{I_r}{kT}} \left(\frac{I_r}{kT} \right)^{-\frac{1}{5}} n_0^5. \quad (10)$$

In the astrophysical examples collected in Table 1 an error of 2-3 units in the evaluation of the n_0 number has little effect on the result, since n_0 is great. However, in our case when n_0 is small an error of even a single unit results in a change in Z_r by an entire order of magnitude, since n_0 enters the formula raised to the fifth power. At the same time, in a real gas n_0 levels will never be achieved for all gas atoms simultaneously. There will be a certain statistical distribution of atoms over n_0 and, generally speaking, the effective value of n_0 will be expressed by a fraction. Below we discuss the me-

thod of evaluating it.

2. Recombination. Now let us examine the phenomenon of recombination. The latter may take place in one of two ways: with the emission of a radiation quantum and as a result of triple collisions, in which case the third particle which transmits the excess energy will almost always be an electron.

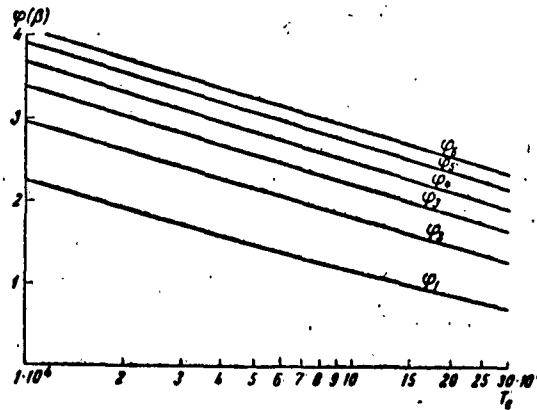


Fig. 4

For the coefficient of recombination with radiation in the hydrogen-similar approximation we can use the following well-known formula [8, 9]:*

$$C_r = 2zA_r \left(\frac{2kT_e}{\pi m_e} \right)^{\frac{1}{2}} \beta \varphi(\beta), \quad (11)$$

where z is the ion charge (in proton-charge units), A_r is the capture constant:

$$A_r = \frac{16}{3\sqrt{3}} \frac{he^2}{m_e^2 c^2} = 2.11 \cdot 10^{-18} \text{ cm}^2; \quad \beta = \frac{I_r}{kT_e} \equiv u_1.$$

$\varphi(\beta)$ is the function presented in tabular form by Spitser [9] and equal to:

$$\varphi(\beta) = \sum_{n=1}^{\infty} \frac{\beta}{n^2} e^{\beta/n^2} \left\{ -\text{Ei} \left(-\frac{\beta}{n^2} \right) \right\}. \quad (12)$$

The quantity $\varphi(\beta)$ for the required interval T_e is of the order of

unity, as can be seen from Fig. 4. Evidently, $\beta/n^2 = u_n$.

The comparative role of the various levels is here determined by the temperature. With an increase in temperature the relative quantity of recombinations at the base level will increase [4]. Therefore the above-mentioned restriction of the number of levels has little effect on recombination with radiation (see the curves in Fig. 5 for the coefficients of recombination with radiation for air).*

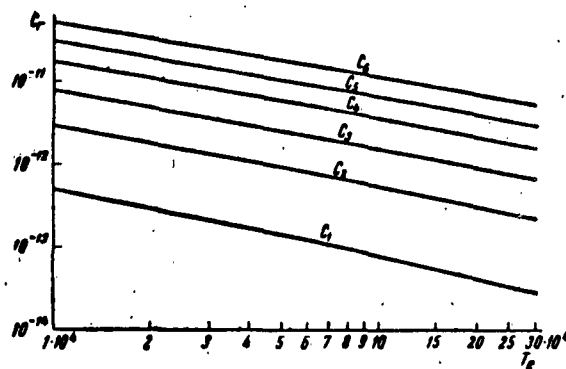


Fig. 5.

For the third recombination (or recombination with triple collisions) in Reference [6] there is derived the formula which determines the probability of recombination at the n level:

$$C_{r,n}^{(3)} = 8.8 \cdot 10^{-21} T^{-3} n^2 e^{u_n} (-\text{Ei}(-u_n)), \quad (13)$$

and after integration with respect to the levels we obtain

$$C_r^{(3)} = 4.15 \cdot 10^{-18} e^{u_1} T^{-3/2} Z_r, \quad (14)$$

or, after substitution of (10) into (14):

$$C_r^{(3)} = 9.13 \cdot 10^{-22} T^{-3} \left(\frac{I_r}{kT} \right)^{-\frac{1}{2}} n_0^2, \quad (15)$$

However, we must, in this case, take into consideration that a portion of the upper levels is not attained as a result of oncoming ionization, and therefore the effective coefficient of triple recombination takes the following form:

$$C_r^* = \sum_n \frac{C_{r,n}^{(3)}}{1 + Z_{r,n} n_e / A_n} \quad (16)$$

where A_n is the probability of spontaneous transition from the n level ($A_n = 1.6 \cdot 10^{-10} n^{-4}$, 5).

The authors of Reference [6] suggest the summing to some level $n_1 < n_0$ beginning from which the terms in Sum (16) sharply diminish, and the units in the denominator can be neglected. The remainder is determined by means of integration from n_1 to n_0 .

For the purposes of our problem, however, this method is not suitable because of the considerations described above. Therefore we will employ the concept of the effective boundary level with the quantum number n_{ef} which we will define as the quantity, which after being substituted into Formula (15) in the place of n_0 , will yield the exact value of $C_{r,1}^*$ calculated according to Formula (16) by direct summation.

As an example we will sum the values of C_r^* in the case of quadruple air ionization ($I_4 = 77$ ev) at $T_e = 10^5$ K, assuming for the time being that all of the levels are "hydrogen-similar" (this is necessary for purposes of illustrating the method).

As we can see from Table 2 the effect of oncoming ionization resulting in the disruption of the upper levels is particularly pronounced in the case of great n_e . The maximum value of C_r^* , with an increase in n_e , transfers from the third level to the second, and then to the base level. The value of n_{ef} for $n_e \geq 10^{20}$ is close to unity or, in other words, the "lifespan" of the upper levels is rather small.

Figure 6 shows the effective coefficient of triple recombination $C^{(3)} n_e$ (solid line) and n_{ef} (dashed lines) to be functions of the electron concentration n_e (1 - hydrogen, $T_e = 17,500^\circ$, $I_r = 13.56$ ev; 2 - air, $T_e = 100,000^\circ$, $I_r = 77$ ev, quadruple ionization). The ratio I_r/kT_e is equal for both of the examples. We obtain the following pattern

(Fig. 6). In the case of rather small concentrations the quantity $c^{(3)}n_e$ increases slowly - there is a transition of the recombination maximum from the upper levels downward, to the base level. Then there is a rapid rise in $c^{(3)}n_e$, since despite the weakening of the role played by the upper levels the oncoming ionization from the base level still does not play any particular role and $c^{(3)}$ remains almost constant as n_e increases. Finally, with a further increase in n_e a saturation sector sets in, and this can be attributed to the oncoming ionization. In this same Fig. 6 is also shown the shape of n_{ef} which approaches unity. With a drop in T , n_{ef} increases approximately as $T^{-1/7}$.

TABLE 2

n.	Z_n	$C_n^{(3)}$	C_n^*		
			$n_e = 10^{18}$	$n_e = 10^{19}$	$n_e = 10^{20}$
1	$4,2 \cdot 10^{-13}$	$8,4 \cdot 10^{-32}$	$8,4 \cdot 10^{-32}$	$8,4 \cdot 10^{-32}$	$8,3 \cdot 10^{-32}$
2	$2,4 \cdot 10^{-9}$	$2,3 \cdot 10^{-30}$	$2,2 \cdot 10^{-30}$	$5,1 \cdot 10^{-31}$	$6,5 \cdot 10^{-32}$
3	$2,3 \cdot 10^{-8}$	$1,4 \cdot 10^{-29}$	$4,7 \cdot 10^{-30}$	$7,0 \cdot 10^{-32}$	$7,0 \cdot 10^{-34}$
4	$7,0 \cdot 10^{-8}$	$4,9 \cdot 10^{-29}$	$2,1 \cdot 10^{-30}$	$2,2 \cdot 10^{-32}$	$2,2 \cdot 10^{-34}$
5	$1,4 \cdot 10^{-7}$	$1,2 \cdot 10^{-28}$	$1,0 \cdot 10^{-30}$	$1,0 \cdot 10^{-32}$	$1,0 \cdot 10^{-34}$
6	$2,2 \cdot 10^{-7}$	$2,6 \cdot 10^{-28}$	$5,8 \cdot 10^{-31}$	$5,8 \cdot 10^{-32}$	$5,8 \cdot 10^{-33}$
$C^* = \sum_{n=1}^{\infty} C_n^*$			$1,3 \cdot 10^{-29}$	$7,2 \cdot 10^{-31}$	$9,1 \cdot 10^{-32}$
n_{ef}			3,23	1,86	1,24

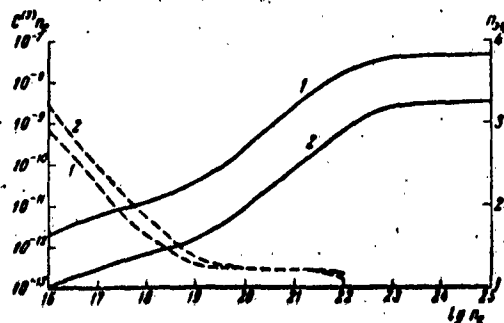


Fig. 6

The comparative role of recombination with radiation and triple

recombination depends on the value of n_e and the magnitude of ionization. For example, at $n_e = 10^{16}$ for the ions NV and OV the triple recombination is 1-2% of the recombination with radiation and it can be neglected. At $n_e = 10^{18}$ it amounts to 7-13%, and at $n_e = 10^{20}$ it amounts to 120% of the recombination with radiation. With transition to the lower ions the role of triple recombination increases.

3. Transition to complex atoms. Up to this point we have regarded all levels in the atoms and ions to be "hydrogen-similar" although in actual fact this is by no means always the case. In order to make the calculation for complex atoms and ions more exact we can use the concept of the effective quantum number n_l^* , determined from the following condition [8, 10]:

$$I_{nl} = I_H \frac{z^2}{n_l^2}, \quad (17)$$

where I_{nl} is the binding energy of the level defined by the quantum numbers n and l ; $I_H = 13.56$ ev represents the potential of ionization for the hydrogen atom; $z = r + 1$ is the charge of the atomic remainder ($z = 1$ for neutral atoms, $z = 2$ for single ions, etc.). The difference $n - n^* = \mu$, referred to as the quantum defect, vanishes with an increase in levels, converting the upper levels into "hydrogen-similar" levels.

In the works of L.M. Biberman and his coworkers [6, 7, 11, 12] there are developed the formulas by means of which we can take into consideration the quantum defect for the actual levels of complex atoms, and these formulas also make it possible to find for these atoms the coefficients of recombination and ionization. The work by Burgess and Seaton [10] presents the general formula for the photoionization section of complex atoms and numerous examples of how this formula can be used.

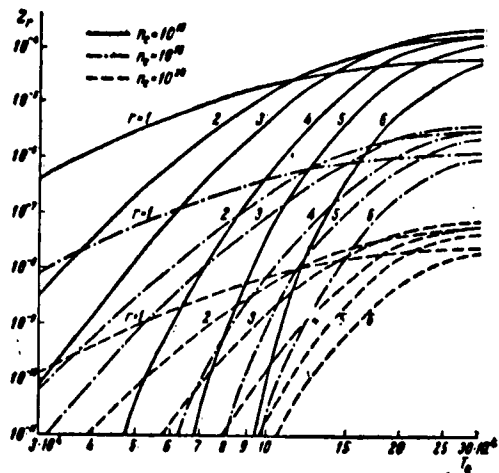


Fig. 7

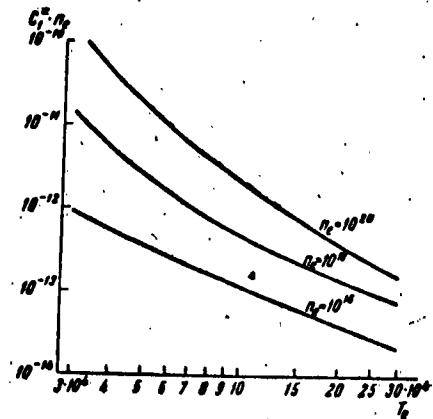


Fig. 8

The formula for the coefficient of recombination with radiation for complex atoms takes the following form:

$$C_r = 1.04 \cdot 10^{-11} T^{-\frac{1}{2}} \int_n^{\infty} \frac{\xi(v, T)}{v} e^{-u} (e^{u'} - 1) dv, \quad (18)$$

where the frequency v is determined by the energy of the recombining electron ($h\nu = mv^2/2$), $u' = h\nu'/kT$, $v' = v$, if $v \leq v_g$ and $v' = v_g$, if $v \geq v_g$, where v_g is the frequency corresponding to the lower excited state n_g from which the integration begins (regarding the selection of n_g and v_g see [11]).

The difference from "hydrogen-similarity" is expressed by the function $\xi(v, T)$, which for various atoms and ions exhibits a varied shape [11]. For atoms of oxygen and nitrogen $\xi(v) < 1$ is not a function of temperature and diminishes almost linearly with v (for $v \leq 10^{-15} \text{ sec}^{-1}$). But for various ions of oxygen the shape of $\xi(v)$ varies substantially, and in this case this quantity may be more or less than unity [12]. However, with an increase in the multiplicity of the ions the shape of $\xi(v)$ is smoothed, approaching the straight line $\xi = 1$, and this is explained by the fact that the levels of ions of high multiplicity approach ever closer to the "hydrogen-similar" levels.

At the high temperatures taking place in the air in the case of shock waves in front of a flying meteorite [$T = (5-20) \cdot 10^4 \text{K}$], the first ionizations take place quickly and do not make a great contribution to the specific heat, whereas the recombination of ions of small multiplicity is negligibly small. Therefore, in this problem the utilization of the exact Formula (18) is not justified by need, and we can assume everywhere that $\xi = 1$. The possible errors for the OI-OIII ions partially compensate each other, since the difference $\xi - 1$ for these ions is opposite in sign.

For our calculations we will, therefore, employ Formulas (11) and (12).

With respect to ionization and triple recombination, the application of Formulas (10) and (16) to the complex atoms need also not result in large errors [6]. However, the factor Γ_r must be introduced into Formula (10), since this factor makes it possible to take into consideration the multiplicity of the atoms:

$$\Gamma_r = \frac{2U_r}{U_{r-i}},$$

where U_r is the sum over the states.

Figures 7 and 8 show the values, respectively, of the coefficients of ionization from all levels and of the coefficients of triple recombination for air, calculated according to the indicated formulas for the electron-temperature interval $T_e = (3-30) \cdot 10^4 \text{K}$.

The author wishes to express his gratitude to K.P. Stanyukovich for his continuous attention to the work, and to L.M. Biberman and G.E. Norman for their useful discussions and the fact that they made it possible for the author to familiarize himself with materials contained in the manuscripts of articles [6, 7]; in addition, the author wishes to express his thanks to S.B. Pikel'ner for his valued advice.

[Footnotes]

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- 16 In Reference [3], in the place of the factor $(1 + 2 (kT_e/I_r))$ there is simply kT_e/I_r and this is, apparently, an error. Therefore the coefficients Z_r are underestimated in this work approximately 0.5 orders of magnitude, which, by the way, has little effect on the results of this work because of the pre-dominant role played by the electron excitation in nebulae.
- 17 In all calculations air was assumed to be a monocomponent gas, and the potentials and coefficients of ionization for nitrogen were averaged in accordance with the relative contents of these gases, i.e., 0.78 and 0.22, respectively.
- 20 In the work by Seaton [8] this formula has a slightly different form, but it is not difficult to modify it to the form of (11). However, this formula is suitable, strictly speaking, only for the isoelectron hydrogen series.
- 21 The question of recombination with radiation for oxygen and nitrogen is also discussed in Reference [13].

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[List of Transliterated Symbols]

22 α_{eff} = α_{eff} = effektivnyy = effective

AERODYNAMIC CHARACTERISTICS OF DELTA WING WITH DEFLECTED AILERON, FLAP, AND OTHER SECONDARY CONTROL SURFACES IN SUPERSONIC FLOW

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(Moscow)

In this work we obtain the aerodynamic characteristics of a delta wing with a deflected aileron, flap, and other secondary control surface, when streamlined by a supersonic flow of gas.

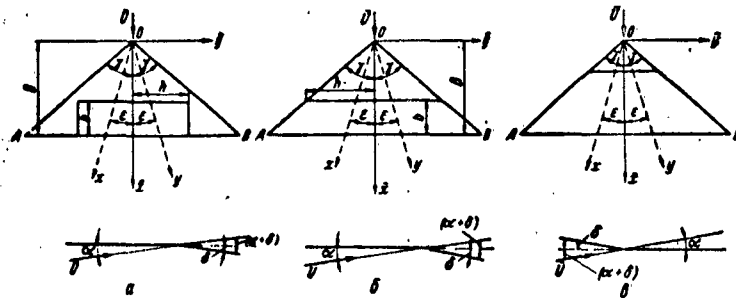


Fig. 1

The theoretical determination of the aerodynamic coefficients is based on the linear theory of a wing of finite span in a supersonic stream, as developed in Reference [1].

1. Statement of the problem. Let us consider the movement of a supersonic gas stream past a thin delta wing with deflected aileron, flap, or other secondary control surface. The wing plane will form a small angle of attack α with the direction of the velocity \bar{U} of the approaching stream. The aileron, flap, or other secondary control surface is deflected from the plane of the wing through the small angle δ (Fig. 1). The leading edges of the wing are supersonic.

Let the flow being formed satisfy the conditions of the linearized theory [1]. Given the indicated streamlining conditions we will determine the aerodynamic characteristics of the wing.

Let us introduce the basic rectangular system of coordinates $\bar{x}\bar{y}\bar{z}$, connected to the wing, and the system of characteristic coordinates $xoyz$ (Fig. 1). The connection between the characteristic coordinates and the basic coordinates is accomplished in accordance with the following formulas known from [1]:

$$x = \bar{x} - k\bar{y}, \quad y = \bar{x} + k\bar{y}, \quad z = k\bar{z}, \quad (1.1)$$

where $k = \sqrt{M^2 - 1} = \cot \varepsilon$; $M > 1$ is the Mach number of the approaching stream and ε is the Mach angle.

The velocity-disturbance potential φ satisfies the wave equation. In accordance with the linear theory for the wing, the boundary conditions for the potential φ in the characteristic coordinates, after linearization, will be the following:

on the projection of the basic part of the wing onto the xoy plane

$$\frac{\partial \varphi}{\partial z} = -U \alpha \operatorname{tg} \varepsilon;$$

on the projection of the aileron, flap, or secondary control surface

$$\frac{\partial \varphi}{\partial z} = -U (\alpha + \delta) \operatorname{tg} \varepsilon. \quad (1.2)$$

Since the boundary conditions (1.2) are constant, the value of the potential φ on the projection of the wing (on the basis of the general formula for the determination of the potential) in the characteristic coordinates is written in the following form:

$$\begin{aligned} \varphi(x, y) = & \frac{U \alpha \operatorname{tg} \varepsilon}{2\pi} \iint_{S_1(x, y)} \frac{d\eta d\xi}{V(x-\xi)(y-\eta)} + \\ & + \frac{U (\alpha + \delta) \operatorname{tg} \varepsilon}{2\pi} \iint_{S_2(x, y)} \frac{d\eta d\xi}{V(x-\xi)(y-\eta)}. \end{aligned} \quad (1.3)$$

where $S_1(x, y)$ is the integration region with respect to the projection of the basic part of the wing falling within the characteristic cone whose apex is situated at the point $M(x, y)$; and $S_2(x, y)$ is the integration region with respect to the aileron, flap, or secondary control surface projection, falling within the same cone.

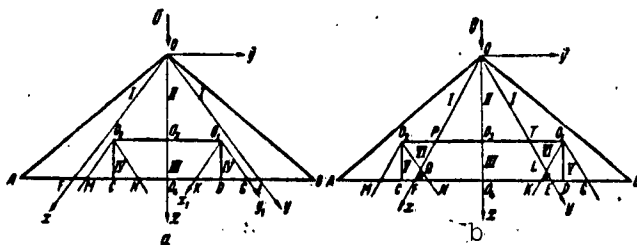


Fig. 2

To determine the forces and moments acting on the wing, it is necessary to know the distribution of the pressure differences across the wing. In accordance with the Bernoulli integral, the difference between the pressure beneath and above the wing is associated with the velocity-disturbance potential φ by the following relationship:

$$\Delta P = -2\rho U (\varphi_x + \varphi_y), \quad (1.4)$$

where ρ is the density of the gas in the undisturbed stream and φ_x and φ_y are the derivatives of the potential φ with respect to the characteristic coordinates.

Thus in order to resolve the posed problem it is necessary to determine the velocity-disturbance potential φ , and then to find the distribution of the pressure difference across the wing.

2. Determination of the pressure difference. a) Distribution of the pressure difference across a wing with an aileron. In flow past a delta wing with a deflected aileron (Fig. 1a) two cases are possible. In the first case (Fig. 2a) the aileron falls completely within the head Mach cone, whereas in the second case (Fig. 2b) the aileron pro-

trudes from the head cone. In both cases the disturbance (perturbation) lines emanating from points O , O_1 , and O_2 divide the entire flow region on the wing into regions I, II, III, IV, V, and VI, and flow in these regions differs in nature. The equations of the wing and aileron generatrices are assumed to be in characteristic coordinates when the potential is sought in these regions.

The flows in regions I and II are known. The distribution of the pressure difference in these regions is determined on the basis of the following formulas:

$$\Delta P_I = 4q\alpha \operatorname{tg} e F(\gamma, e), \quad (2.1)$$

$$\Delta P_{II} = 4q\alpha \operatorname{tg} e F(\gamma, e) \Phi_1(\gamma, e, \varphi), \quad (2.2)$$

where we have denoted

$$F(\gamma, e) = \frac{\operatorname{tg} \gamma}{\sqrt{\operatorname{tg}^2 \gamma - \operatorname{tg}^2 e}}, \quad (2.3)$$

$$\Phi_1(\gamma, e, \varphi) = \frac{1}{\pi} \arccos \left[1 - 2 \left(\frac{\operatorname{tg}^2 \gamma - \operatorname{tg}^2 e}{\operatorname{tg}^2 \gamma - \operatorname{tg}^2 \varphi} \right) \right], \quad (2.4)$$

φ is the polar angle of the point in region II counted off from the \overline{ox} axis, and here $\varphi > 0$ in the region O_4OE and $\varphi < 0$ in the region O_4OF ; q is the velocity head.

Using Formulas (1.3) and (1.4), we obtain the following expression for the pressure difference in region III:

$$\Delta P_{III} = 4q\delta \operatorname{tg} e + 4q\alpha \operatorname{tg} e F(\gamma, e) \Phi_1(\gamma, e, \varphi), \quad (2.5)$$

where φ is the polar angle of the point in region III counted off from the \overline{ox} axis.

In seeking the potential $\varphi_{IV}(x, y)$ in region IV, we do not take into consideration the effect of the slit, but we do take into consideration the leakage of the gas from one half of the region to the other. Integration in the region IV is more conveniently carried out in characteristic coordinates whose origin is situated at the point

O_1 . For the pressure difference in region IV we obtain the following formula:

$$\begin{aligned} \Delta P_{IV} = & 4q\delta \operatorname{tg} \varepsilon \frac{2}{\pi} \operatorname{arc} \operatorname{tg} \sqrt{\frac{x_1}{y_1}} + \\ & + 4q\alpha \operatorname{tg} \varepsilon F(\gamma, \varepsilon) \frac{2}{\pi} \left\{ \operatorname{arc} \operatorname{tg} \sqrt{\frac{m[x_1 + (B-b) - kh]}{[y_1 + (B-b) + kh]}} + \right. \\ & \left. + \operatorname{arc} \operatorname{ctg} \sqrt{\frac{n[x_1 + (B-b) - kh]}{[y_1 + (B-b) + kh]}} \right\}, \end{aligned} \quad (2.6)$$

where

$$m = \frac{\operatorname{tg} \gamma - \operatorname{tg} \varepsilon}{\operatorname{tg} \gamma + \operatorname{tg} \varepsilon}, \quad n = \frac{1}{m}. \quad (2.7)$$

x_1 and y_1 are the characteristic coordinates of the point (the origin is at point O_1) in region IV; B , b , and h are the geometric parameters shown in Fig. 1.

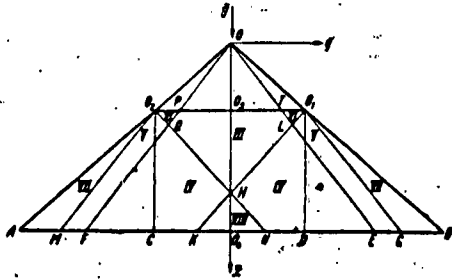


Fig. 3

The connection between the characteristic coordinates (x, y) whose origin is situated at point O and the characteristic coordinates (x_1, y_1) whose origin is situated at point O_1 is achieved by the following formulas:

$$x = x_1 + (B-b) - kh, \quad y = y_1 + (B-b) + kh. \quad (2.8)$$

On the basis of (2.8) Formula (2.6) can be rewritten in the following form:

$$\begin{aligned} \Delta P_{IV} = & 4q\delta \operatorname{tg} \varepsilon \frac{2}{\pi} \operatorname{arc} \operatorname{tg} \sqrt{\frac{x_1}{y_1}} + \\ & + 4q\alpha \operatorname{tg} \varepsilon F(\gamma, \varepsilon) \frac{2}{\pi} \left\{ \operatorname{arc} \operatorname{tg} \sqrt{\frac{mx}{y}} + \operatorname{arc} \operatorname{ctg} \sqrt{\frac{nx}{y}} \right\}. \end{aligned} \quad (2.9)$$

Rearranging and introducing the denotation

$$\Phi_{\varepsilon(\varepsilon, \varphi_1)} = \frac{2}{\pi} \operatorname{arc} \operatorname{tg} \sqrt{\frac{\operatorname{tg} \varepsilon - \operatorname{tg} \varphi_1}{\operatorname{tg} \varepsilon + \operatorname{tg} \varphi_1}}, \quad (2.10)$$

for the pressure difference in region IV we will obtain the final expression

$$\Delta P_{IV} = 4q\delta \operatorname{tg} \varepsilon \Phi_{\varepsilon}(\varepsilon, \varphi_1) + 4q\alpha \operatorname{tg} \varepsilon \Phi_{\varepsilon}(\gamma, \varepsilon, \varphi), \quad (2.11)$$

where φ_1 is the polar angle of the point in region IV, counted off from O_1D (the pole is situated at point O_1), and $\varphi_1 > 0$ in the region O_1DG ; $\varphi_1 < 0$ in the region O_1DK ; φ is the polar angle of the same point in region IV, counted off from the $O\bar{x}$ axis (the pole is situated at point O). For the remaining regions the pressure difference is determined by the following formulas:

$$\Delta P_V = 4q\alpha \operatorname{tg} \epsilon F(\gamma, \epsilon) + 4q\delta \operatorname{tg} \epsilon \Phi_2(\epsilon, \varphi_1), \quad (2.12)$$

$$\Delta P_{VI} = 4q\alpha \operatorname{tg} \epsilon F(\gamma, \epsilon) + 4q\delta \operatorname{tg} \epsilon \quad (2.13)$$

taking into consideration (2.3) and (2.4).

b) Distribution of pressure difference across a wing with flap and secondary control surface. In the case of flow past a delta wing with a flap or secondary control surface the entire flow region is divided into eight characteristic regions I, II, III, IV, V, VI, VII, and VIII by perturbation (disturbance) lines emanating from points O , O_1 , and O_2 (Fig. 3) in the general case.

In regions I and II of a wing with a flap, the pressure difference is determined according to Formulas (2.1) and (2.2), taking into consideration (2.3) and (2.4). For a wing with a secondary control surface we will have

$$\begin{aligned} \Delta P_I &= 4q(\alpha + \delta) \operatorname{tg} \epsilon F(\gamma, \epsilon), \\ \Delta P_{II} &= 4q(\alpha + \delta) \operatorname{tg} \epsilon F(\gamma, \epsilon) \times \end{aligned} \quad (2.14)$$

$$\times \Phi_1(\gamma, \epsilon, \varphi). \quad (2.15)$$

The pressure difference in region VII of a wing with a flap is determined according to Formula (2.14) and according to Formula (2.1) in the case of a wing with a secondary control surface. In region III of a wing with a flap the pressure difference is determined on the basis of Formula (2.5).

For a wing with a secondary control surface, we will obtain

$$\Delta P_{III} = 4q(\alpha + \delta) \operatorname{tg} \varepsilon F(\gamma, \varepsilon) \Phi_1(\gamma, \varepsilon, \varphi) - 4q\delta \operatorname{tg} \varepsilon. \quad (2.16)$$

In region VI of a wing with a flap the pressure difference is determined according to Formula (2.13). For a wing with a secondary control surface we will have

$$\Delta P_{VI} = 4q(\alpha + \delta) \operatorname{tg} \varepsilon F(\gamma, \varepsilon) - 4q\delta \operatorname{tg} \varepsilon. \quad (2.17)$$

For the pressure difference in the region V of a wing with a flap we obtain the expression

$$\Delta P_V = 4q\alpha \operatorname{tg} \varepsilon F(\gamma, \varepsilon) + 4q\delta \operatorname{tg} \varepsilon (\Phi_2(\varepsilon, \varphi_1) + F(\gamma, \varepsilon) [1 - \Phi_2(\gamma, \varepsilon, \varphi_1)]) \quad (2.18)$$

where

$$\Phi_2(\gamma, \varepsilon, \varphi_1) = \frac{2}{\pi} \operatorname{arc} \operatorname{tg} \sqrt{\frac{(\operatorname{tg} \gamma + \operatorname{tg} \varepsilon)(\operatorname{tg} \varepsilon - \operatorname{tg} \varphi_1)}{(\operatorname{tg} \gamma - \operatorname{tg} \varepsilon)(\operatorname{tg} \varepsilon + \operatorname{tg} \varphi_1)}} \quad (2.19)$$

φ_1 is the polar angle of the point in region V (the pole is situated at point O_1). For a wing with a secondary control surface we will have

$$\Delta P_V = 4q(\alpha + \delta) \operatorname{tg} \varepsilon F(\gamma, \varepsilon) - 4q\delta \operatorname{tg} \varepsilon (\Phi_2(\varepsilon, \varphi_1) + F(\gamma, \varepsilon) [1 - \Phi_2(\gamma, \varepsilon, \varphi_1)]). \quad (2.20)$$

For the pressure differences in regions IV and VIII of a wing with a flap we obtain the following formulas:

$$\begin{aligned} \Delta P_{IV} = & 4q\alpha \operatorname{tg} \varepsilon F(\gamma, \varepsilon) \Phi_1(\gamma, \varepsilon, \varphi) + \\ & + 4q\delta \operatorname{tg} \varepsilon (\Phi_2(\varepsilon, \varphi_1) + F(\gamma, \varepsilon) [1 - \Phi_2(\gamma, \varepsilon, \varphi_1)]), \end{aligned} \quad (2.21)$$

$$\begin{aligned} \Delta P_{VIII} = & 4q\alpha \operatorname{tg} \varepsilon F(\gamma, \varepsilon) \Phi_1(\gamma, \varepsilon, \varphi) + 4q\delta \operatorname{tg} \varepsilon (\Phi_2(\varepsilon, \varphi_1) + F(\gamma, \varepsilon) [1 - \Phi_2(\gamma, \varepsilon, \varphi_1)] + \\ & + [\Phi_2(\varepsilon, \varphi_2) - 1] + F(\gamma, \varepsilon) [1 - \Phi_2(\gamma, \varepsilon, \varphi_2)]), \end{aligned} \quad (2.22)$$

where φ is the polar angle of the point in region VIII, counted off from the ox axis (the pole is situated at point O), φ_1 is the polar angle of the same point in region VIII, counted off from the line O_1D (the pole is situated at point O_1), φ_2 is the polar angle of the same point in region VIII, counted off from the line O_2C (the pole is situated at point O_2), and $\varphi_2 > 0$ in the region O_2CM and $\varphi_2 < 0$ in the region O_2CN .

In order to derive the formulas for the pressure difference in regions IV and VIII of a wing with a secondary control surface, it is

sufficient to replace α by $(\alpha + \delta)$ and δ by $-\delta$ in Formulas (2.21) and (2.22), which follows from an analysis of the formulas derived above.

3. Determination of forces and moments acting on the wing. The forces and moments acting on a wing are determined by integration with respect to all regions on the wing, and this is most conveniently carried out in polar coordinates.

a) Characteristics of a wing with an aileron. Since in all of the formulas for the determination of the pressure difference in various regions of a wing with an aileron there are terms that are functions only of α , we will separately calculate the force acting on the wing and determined by these terms. In either case (Fig. 2a and b) we will obtain

$$Z_1 = 4q\alpha \operatorname{tg} \epsilon B^2 \operatorname{tg} \gamma. \quad (3.1)$$

Using Formulas (2.5) and (2.13) we will find the force acting on the region $O_1O_2O_4K$ and dependent on δ . Carrying out the calculation, we will obtain

$$Z'_1 = 4q\delta \operatorname{tg} \epsilon \left(bh - \frac{b^2}{2} \operatorname{tg} \epsilon \right). \quad (3.2)$$

The exact same force acts on region $O_2O_3O_4N$. On the basis of (2.11) and (2.12), with (2.10) taken into consideration, we will determine the force acting on region KO_1G and dependent on δ . Carrying out the integration, we will find

$$Z''_1 = 4q\delta \operatorname{tg} \epsilon \frac{b^2}{2} \operatorname{tg} \epsilon. \quad (3.3)$$

The force acting on the region GO_1O_2M and dependent on δ is determined from the following formula:

$$Z_3 = 4q\delta \operatorname{tg} \epsilon 2bh. \quad (3.4)$$

For the total force acting on the wing we will obtain the following expression:

$$Z = 4q\alpha \operatorname{tg} \epsilon B^2 \operatorname{tg} \gamma + 4q\delta \operatorname{tg} \epsilon 2bh. \quad (3.5)$$

The coefficient for this force has the following form:

$$\frac{C_z \sqrt{M^2 - 1}}{4\alpha} = \left(1 + \frac{\delta}{\alpha} \frac{S_e}{S_k}\right). \quad (3.6)$$

where $S_e = 2bh$ is the total aileron area, $S_k = B^2 \tan \gamma$ is the total area of the wing.

Let us determine the moment of forces with respect to the \bar{oy} axis. The moment of forces (3.1) is determined in accordance with the following formula:

$$M_{y1} = Z_1 \frac{2}{3} B. \quad (3.7)$$

For the moment of forces (3.4) we obtain the following expression:

$$M_{y2} = 4q\delta \operatorname{tg} \epsilon 2bh \left(B - \frac{b}{2}\right). \quad (3.8)$$

The total moment of all forces with respect to the \bar{oy} axis takes the following form:

$$M_y = 4q\alpha \operatorname{tg} \epsilon \frac{2}{3} B^2 \operatorname{tg} \gamma + 4q\delta \operatorname{tg} \epsilon 2Bbh \left(1 - \frac{b}{2B}\right). \quad (3.9)$$

For the coefficient of this moment we obtain the following formula:

$$\frac{m_y \sqrt{M^2 - 1}}{4\alpha} = \frac{2}{3} + \frac{\delta}{\alpha} \left(1 - \frac{b}{2B}\right) \frac{S_e}{S_k}. \quad (3.10)$$

Dividing (3.10) by (3.6), we will obtain the dimensionless coordinate for the center of pressure of the central chord. Carrying out this division, we will obtain

$$\bar{x}_{H,x} = \frac{2}{3} + \Delta \bar{x}_{H,x}, \quad (3.11)$$

where

$$\Delta \bar{x}_{u, \alpha} = \frac{\frac{\delta}{\alpha} \left(\frac{1}{3} - \frac{b}{2B} \right) \frac{S_2}{S_1}}{\left(1 + \frac{\delta}{\alpha} \frac{S_2}{S_1} \right)} \quad (3.12)$$

Let us determine the moment of forces acting on one half of the wing, about the $\bar{o}\bar{x}$ axis. The total moment of forces acting on this half of the wing, about the $\bar{o}\bar{x}$ axis, takes the following form

$$M_{\bar{x}} = 4q\alpha \operatorname{tg} \epsilon \frac{1}{3} B^2 \operatorname{tg}^2 \gamma \frac{1}{\pi} \left[\frac{\operatorname{tg} \epsilon}{\operatorname{tg} \gamma} + \frac{\operatorname{tg} \gamma}{\sqrt{\operatorname{tg}^2 \gamma - \operatorname{tg}^2 \epsilon}} \arccos \frac{\operatorname{tg} \epsilon}{\operatorname{tg} \gamma} \right] + \quad (3.13)$$

$$+ 4q\delta \operatorname{tg} \epsilon \frac{1}{2} b h^2 \left[1 + \frac{1}{6} \left(\frac{b}{h} \right)^2 \operatorname{tg}^2 \epsilon \right].$$

For the coefficient of this moment we will obtain the following expression:

$$\frac{m_{\bar{x}} \sqrt{M^2 - 1}}{4\alpha} = \frac{1}{3} \frac{2}{\pi} \left[\frac{\operatorname{tg} \epsilon}{\operatorname{tg} \gamma} + \frac{\operatorname{tg} \gamma}{\sqrt{\operatorname{tg}^2 \gamma - \operatorname{tg}^2 \epsilon}} \arccos \frac{\operatorname{tg} \epsilon}{\operatorname{tg} \gamma} \right] + \quad (3.14)$$

$$+ \frac{\delta}{\alpha} \frac{1}{2} \left(1 + \frac{1}{6} \lambda^2 \operatorname{tg}^2 \epsilon \right) \frac{h}{H} \frac{S_2}{S_1},$$

where $S_2 = bh$ is the area of one half of the aileron, $S_1 = (B^2/2) \tan \gamma$ is the area of one half of the wing, and $H = B \tan \gamma$ is the half-span of the wing; $\lambda = b/h$.

Formula (3.13) can be used for the determination of the moment of forces of a rotating wing or control surface about the side chord in the case of flow past a wall. At $\alpha = 0$ we will obtain the moment produced by the deflection of the aileron.

The dimensionless coordinate of the center of pressures of the forces acting on the half of the wing, about the half-span, is found from the following expression:

$$\bar{y}_{u, \alpha} = \frac{m_{\bar{y}}}{c_x} \quad (3.15)$$

b) Characteristic of a wing with a flap or secondary control surface. Let us determine the forces and moments acting on a wing with a flap. Using the formulas for the pressure difference, obtained for a

wing with a flap, we will, through integration, find the force

$$Z = 4q\alpha \operatorname{tg} \epsilon B^2 \operatorname{tg} \gamma + 4q\delta \operatorname{tg} \epsilon [2bh + b^2 \operatorname{tg} \gamma]. \quad (3.16)$$

For the coefficient of this force we will obtain the expression

$$\frac{c_z \sqrt{M^2 - 1}}{4\alpha} = \left(1 + \frac{\delta}{\alpha} \frac{S_z}{S_K}\right), \quad (3.17)$$

where $S_z = (2bh + b^2 \tan \gamma)$ is the area of the flap.

The moment of force (3.16) about the $\bar{o}\bar{y}$ axis takes the following form:

$$M_y = 4q\alpha \operatorname{tg} \epsilon \frac{2}{3} B^2 \operatorname{tg} \gamma + 4q\delta \operatorname{tg} \epsilon (2bh + b^2 \operatorname{tg} \gamma) \left(B - \frac{b}{3}\right) - 4q\delta \operatorname{tg} \epsilon \frac{1}{3} b^2 h. \quad (3.18)$$

For the coefficient of the moment (3.18) we obtain the following formula:

$$\frac{m_y \sqrt{M^2 - 1}}{4\alpha} = \frac{2}{3} + \frac{\delta}{\alpha} \left[\left(1 - \frac{b}{3B}\right) \frac{S_z}{S_K} - \frac{b}{6B} \frac{S_{z,n}}{S_K} \right], \quad (3.19)$$

where $S_{z,p} = 2bh$.

Dividing (3.19) by (3.17), we will obtain

$$\bar{x}_{u,n} = \frac{2}{3} + \Delta \bar{x}_{u,n}, \quad (3.20)$$

where

$$\Delta \bar{x}_{u,n} = \frac{\frac{\delta}{\alpha} \left[\frac{1}{3} \left(1 - \frac{b}{B}\right) \frac{S_z}{S_K} - \frac{b}{6B} \frac{S_{z,n}}{S_K} \right]}{\left(1 + \frac{\delta}{\alpha} \frac{S_z}{S_K}\right)}. \quad (3.21)$$

The total moment of forces acting on one half of the wing about the central chord, takes the following form:

$$M_x = 4q\alpha \operatorname{tg} \epsilon \frac{1}{3} B^2 \operatorname{tg}^2 \gamma \frac{1}{\pi} \left[\frac{\operatorname{tg} \epsilon}{\operatorname{tg} \gamma} + \frac{\operatorname{tg} \gamma}{\sqrt{\operatorname{tg}^2 \gamma - \operatorname{tg}^2 \epsilon}} \arccos \frac{\operatorname{tg} \epsilon}{\operatorname{tg} \gamma} \right] + \\ + 4q\delta \operatorname{tg} \epsilon h \left\{ \frac{bh}{2} \left[1 + \frac{1}{\theta} \left(\frac{b}{h}\right)^2 \operatorname{tg}^2 \epsilon \right] + \frac{b^2 \operatorname{tg} \gamma}{2} \left[1 + \frac{1}{3} \left(\frac{b}{h}\right) \operatorname{tg} \gamma \right] \right\}. \quad (3.22)$$

For the coefficient of this moment we obtain the following expression:

$$\frac{m_x \sqrt{M^2 - 1}}{4\alpha} = \frac{1}{3} \frac{2}{\pi} \left[\frac{\operatorname{tg} \epsilon}{\operatorname{tg} \gamma} + \frac{\operatorname{tg} \gamma}{\sqrt{\operatorname{tg}^2 \gamma - \operatorname{tg}^2 \epsilon}} \arccos \frac{\operatorname{tg} \epsilon}{\operatorname{tg} \gamma} \right] +$$

$$+ \frac{\delta}{\alpha} \frac{h}{H} \left[\frac{1}{2} \left(1 + \frac{1}{6} \lambda^2 \operatorname{tg}^2 \gamma \right) \frac{S_1}{S_1} + \left(1 + \frac{1}{3} \lambda \operatorname{tg} \gamma \right) \frac{S_1}{S_1} \right],$$

where we denote

$$S_1 = \frac{B^2}{2} \operatorname{tg} \gamma, \quad S_2 = bh, \quad S_3 = \frac{b^2}{2} \operatorname{tg} \gamma, \quad H = B \operatorname{tg} \gamma, \quad \lambda = \frac{b}{h}.$$

The dimensionless coordinate of the center of pressure of the forces acting on the half wing about the half-span can be found from Formula (3.15).

In order to determine the aerodynamic coefficients of a wing with a secondary control surface it is necessary to replace α by $(\alpha + \delta)$ and δ by $-\delta$ in the formulas found for a wing with a flap. Carrying out this operation in Formula (3.16), we will obtain

$$Z = 4q(\alpha + \delta) \operatorname{tg} \epsilon B^2 \operatorname{tg} \gamma - 4q\delta \operatorname{tg} \epsilon (2bh + b^2 \operatorname{tg} \gamma). \quad (3.24)$$

Hence we find that

$$Z = 4q\alpha \operatorname{tg} \epsilon S_p + 4q\delta \operatorname{tg} \epsilon S_n, \quad (3.25)$$

where $S_p = [B^2 \tan \gamma - (2bh + b^2 \tan \gamma)]$ is the area of the secondary control surface.

For the coefficient of force (3.25) we will obtain the following expression:

$$\frac{c_z \sqrt{M^2 - 1}}{4\alpha} = \left(1 + \frac{\delta}{\alpha} \frac{S_n}{S_p} \right). \quad (3.26)$$

Using Formula (3.18), we will find

$$M_y = 4q\alpha \operatorname{tg} \epsilon B^2 \operatorname{tg} \gamma \frac{2}{3} B + 4q\delta \operatorname{tg} \epsilon \left\{ \left[B^2 \operatorname{tg} \gamma - (2bh + b^2 \operatorname{tg} \gamma) \right] \frac{2}{3} B - (2bh + b^2 \operatorname{tg} \gamma) \frac{1}{3} (B - b) + \frac{1}{3} b^2 h \right\}. \quad (3.27)$$

For the coefficient of the moment (3.27) we will obtain the following expression:

$$\frac{m_y \sqrt{M^2 - 1}}{4\alpha} = \frac{2}{3} + \frac{\delta}{\alpha} \left[\frac{2}{3} \frac{S_n}{S_p} - \frac{1}{3} \left(1 - \frac{b}{B} \right) \frac{S_3}{S_p} + \frac{1}{6} \frac{b}{B} \frac{S_{3,n}}{S_p} \right]. \quad (3.28)$$

The formula for the determination of $\bar{x}_{ts,d}$ takes the following form:

$$\bar{x}_{u, \lambda} = \frac{2}{3} - \Delta \bar{x}_{u, \lambda} \quad (3.29)$$

where

$$\Delta \bar{x}_{u, \lambda} = \frac{\frac{\delta}{\alpha} \left[\frac{1}{3} \left(1 - \frac{b}{B} \right) \frac{S_3}{S_\kappa} - \frac{1}{6} \frac{S_{3, \pi}}{S_\kappa} \frac{b}{B} \right]}{\left(1 + \frac{\delta}{\alpha} \frac{S_\pi}{S_\kappa} \right)} \quad (3.30)$$

Carrying out the substitution in (3.22), we will obtain

$$M_x = 4q(\alpha + \delta) \operatorname{tg} \epsilon \frac{1}{3} B^3 \operatorname{tg}^2 \gamma \frac{1}{\pi} \left[\frac{\operatorname{tg} \epsilon}{\operatorname{tg} \gamma} + \frac{\operatorname{tg} \gamma}{\sqrt{\operatorname{tg}^2 \gamma - \operatorname{tg}^2 \epsilon}} \arccos \frac{\operatorname{tg} \epsilon}{\operatorname{tg} \gamma} \right] - \quad (3.31)$$

$$- 4q\delta \operatorname{tg} \epsilon h \left\{ \frac{bh}{2} \left[1 + \frac{1}{6} \left(\frac{b}{h} \right)^2 \operatorname{tg}^2 \epsilon \right] + \frac{b^3}{2} \operatorname{tg} \gamma \left[1 + \frac{1}{3} \left(\frac{b}{h} \right) \operatorname{tg} \gamma \right] \right\}.$$

For the coefficient of the moment (57) we will obtain the following expression:

$$\frac{m_x \sqrt{M^2 - 1}}{4\alpha} = \left(1 + \frac{\delta}{\alpha} \right) \frac{1}{3} \frac{2}{\pi} \left[\frac{\operatorname{tg} \epsilon}{\operatorname{tg} \gamma} + \frac{\operatorname{tg} \gamma}{\sqrt{\operatorname{tg}^2 \gamma - \operatorname{tg}^2 \epsilon}} \arccos \frac{\operatorname{tg} \epsilon}{\operatorname{tg} \gamma} \right] - \quad (3.32)$$

$$- \frac{\delta}{\alpha} \frac{h}{H} \left[\frac{1}{2} \left(1 + \frac{1}{6} \lambda^2 \operatorname{tg}^2 \epsilon \right) \frac{S_2}{S_1} + \left(1 + \frac{1}{3} \lambda \operatorname{tg} \gamma \right) \frac{S_4}{S_1} \right].$$

Dividing (3.32) by (3.26), we will find the dimensionless coordinate of the center of pressure of the forces acting on the half wing about the half-span.

According to the found formulas for c_z and $\bar{x}_{ts, d}$ we carried out the calculations which were then compared against the experimental results. The comparisons showed that the calculated data are in good agreement with the experimental within the following range of numbers: $2 \leq M \leq 4.1$.

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[List of Transliterated Symbols]

- 37 $u, d = ts, d = tsentr \text{ davleniya} = \text{center of pressure}$
- 37 $\epsilon = e = eleron = \text{aileron}$
- 37 $\kappa = k = krylo = \text{wing}$

39 $z = z = \text{zakrylok} = \text{flap}$

39 $z.n = z.p = \text{zakrylok predkrylok} = \text{flap and other control surface}$

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